Semi-analytical formulas for the fundamental parameters of Galactic early B supergiants

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January 13, 2010

The publication of new tables of calibration of some fundamental parameters of Galactic B0-B5 supergiants in the two classes Ia and Ib allow to particularize the eight parameters conjecture that model five fundamental parameters. The numerical expression for visual magnitude ,radius , mass , luminosity and surface gravity are derived for supergiants in the range of temperature between 29700 and 15200. The presence of accurate tables of calibration allows us to introduce the efficiency of the derived formulas. The average efficiency of the new formulas ,expressed in percent, is 94 for the visual magnitude, 81 for the mass , 96 for the radius , 99 for the logarithm of the luminosity and 97 for the logarithm of the surface gravity.
keywords
stars: formation ; stars: statistics ; methods: data analysis ; techniques: photometric

1 Introduction

A recent study on the supergiants of spectral type B0-B5 , see Searle et al. (2008) , reports a fine grid of the fundamental parameters such as the tem-

perature , luminosity , radius, surface gravity and mass. In this paper we first review the four fundamental parameters as modeled by the eight parameters conjecture and we add to the list the surface gravity, see Section 2. Section 3 contains the numerical expression for five fundamental parameters as well as the efficiency of such a evaluation.

2 The fundamental formulas

We briefly review the formulas that characterize the visual magnitude,M_V , the mass ,$\mathcal{M}$, the radius ,R, and the luminosity , L, of the stars for each MK class as function of the intrinsic , unreddened color index $(B-V)_0$, see Zaninetti (2008) for details. The first equation models the visual magnitude , M_V

$$\begin{gathered} M_V = -2.5\, a_{\rm LM} - 2.5\, b_{\rm LM}\, a_{\rm MT} - \\ 2.5\, b_{\rm LM}\, b_{\rm MT}\, \mathit{log_{10}}(\frac{T_{\rm BV}}{(B-V)_{\mathit{0}} - K_{\rm BV}}) \\ -K_{\rm BC} + 10\, \mathit{log_{10}}(\frac{T_{\rm BV}}{(B-V)_{\mathit{0}} - K_{\rm BV}}) + \\ \frac{T_{\rm BC}}{T_{\rm BV}}\, [(B-V)_0 - K_{\rm BV}] + M_{{\rm bol},\odot} \quad . \end{gathered} \tag{1}$$

The second equation connects the mass of the star ,$\mathcal{M}$, with $(B-V)_0$

$$\begin{gathered} \log_{10}(\frac{\mathcal{M}}{\mathcal{M}_\odot}) = a_{MT} + \\ b_{MT}\, \ln\left(\frac{T_{BV}}{(B-V)_{\mathit{0}} - K_{BV}}\right) (\ln(10))^{-1} \quad , \end{gathered} \tag{2}$$

where $\mathcal{M}_\odot$ is the sun's mass. The third equation regulates the radius R, with $(B-V)_0$

$$\log_{10}(\frac{R}{R_\odot}) =$$
$$1/2\, a_{LM} + 1/2\, b_{LM}\, a_{MT} + 2\, \frac{\ln\left(T_\odot\right)}{\ln(10)} +$$

$$+1/2\, b_{LM}\, b_{MT}\, \ln\left(\frac{T_{BV}}{(B-V)_0 - K_{BV}}\right)(\ln(10))^{-1}$$
$$-2\, \ln\left(\frac{T_{BV}}{(B-V)_0 - K_{BV}}\right)(\ln(10))^{-1} \quad , \tag{3}$$

where $R\odot$ is the sun's radius. The fourth equation connects the luminosity of a star L with $(B-V)_0$

$$\log_{10}(\frac{L}{L\odot}) = a_{LM} + b_{LM}\, a_{MT}$$
$$+b_{LM}\left(b_{MT}\, \ln\left(\frac{T_{BV}}{(B-V)_0 - K_{BV}}\right)\frac{1}{\ln(10)}\right) \; , \tag{4}$$

where $L\odot$ is the sun's luminosity. The eight numerical parameters that compare above are reported in Table 1 as well as the physical or empirical formula that regulates them.

Table 1: Synoptic Table of the eight coefficients, BC is the bolometric correction

Coefficient	adopted relationship
$K_{\rm BV}$,$T_{\rm BV}$[K]	$(B-V)_0 = K_{\rm BV} + T_{\rm BV}/T$
$K_{\rm BC}$,$T_{\rm BC}$[K]	$BC = -\frac{T_{\rm BC}}{T} - 10\ \log_{10}\ T + K_{\rm BC}$
$a_{\rm LM}$, $b_{\rm LM}$	$\log_{10}(L/L_{\odot}) = a_{\rm LM} + b_{\rm LM} \log_{10}(\mathcal{M}/\mathcal{M}_{\odot})$
$a_{\rm MT}$, $b_{\rm MT}$	$\log_{10}(\mathcal{M}/\mathcal{M}_{\odot}) = a_{\rm MT} + b_{\rm MT} \log_{10}(T/T_{\odot})$

A fifth fundamental parameter is the surface gravity , g , that is defined as

$$g = G\frac{M}{r^2} \quad , \tag{5}$$

where M is the mass of the body , r its radius and G is the Newtonian gravitational constant which has value $G = 6.6742 \times 10^{-11} \frac{m^3}{kgs^2}$, Mohr & Taylor (2005). On adopting $R_{sun} = 6.95508 \cdot 10^8\ m$ and $M_{sun} = 1.989 \cdot 10^{30}\ kg$, see Cox (2000), we obtain the following expression for the logarithm of the surface gravity

$$log(g[cgs]) = -10.60 + 0.4342 \times$$
$$\times \ln\left(e^{2.302\, a_{MT} - 2.302\, a_{LM} - 2.302\, b_{LM}\, a_{MT}}\left(\frac{T_{BV}}{BV - K_{BV}}\right)^{b_{MT} - b_{LM}\, b_{MT} + 4}\right) \quad . \tag{6}$$

3 Application to the supergiants

The eight parameters conjecture may represents an acceptable fit of five fundamental parameters of the stars once the calibration data are available in the considered MK class, in our case B0-B5 supergiants. The Table 5 in Searle et al. (2008) provides the calibration of $\log_{10}(\frac{L}{L\odot})$, $\log_{10}(\frac{R}{R\odot})$, $\frac{\mathcal{M}}{\mathcal{M}\odot}$ and $\log_{10}(g[cm/s^2])$ as function of the temperature T in the range $29700 < T < 15200$. Table 2 reports the eight parameters as well as the source where the calibrated data resides.

Table 2: Table of the adopted coefficients for B0-B5 supergiants .

Coefficient	Ia	Ib	source of data
$K_{\rm BV}$	-0.3961	-0.3961	Table 15.7 in Cox (2000)
$T_{\rm BV}$[K]	4011.7	4011.7	Table 15.7 in Cox (2000)
$K_{\rm BC}$	42.87	42.87	Table 15.7 in Cox (2000)
$T_{\rm BC}$[K]	31573.8	31573.8	Table 15.7 in Cox (2000)
$a_{\rm LM}$	4.667	4.092	Table 5 in Searle et al. (2008)
$b_{\rm LM}$	0.6050	0.92054	Table 5 in Searle et al. (2008)
$a_{\rm MT}$	-3.713	-6.0246	Table 5 in Searle et al. (2008)
$b_{\rm MT}$	1.1674	1.7213	Table 5 in Searle et al. (2008)

The fundamental parameters of the stars are parametrized according to the MK class to which they belong and the intrinsic , unreddened color index $(B-V)_0$ or the temperature as derived , for example , from spectroscopic arguments. The conversion between temperature and $(B-V)_0$ is obtained trough the following two formulas

$$(B-V)_0 = -0.3961 + \frac{4011.7}{T} \qquad (7)$$
$$15200\ K < T < 30000\ K \quad ,$$

$$T = \frac{4011.7}{(B-V)_0 + 0.3961} \qquad (8)$$
$$-0.25 < (B-V)_0 < -0.14 \quad .$$

From a numerical point of view the visual magnitude , $M_{\rm V}$, is

$$M_V = -41.07 + 3.576\ \ln\left(4012.0\ \left((B-\ V)_0 + 0.3961\right)^{-1}\right) + 7.870\,(B-\ V)_0 \qquad (9)$$
$$supergiants\ Ia\ when\ 0.25 < (B-V)_0 < -0.14 \quad ,$$

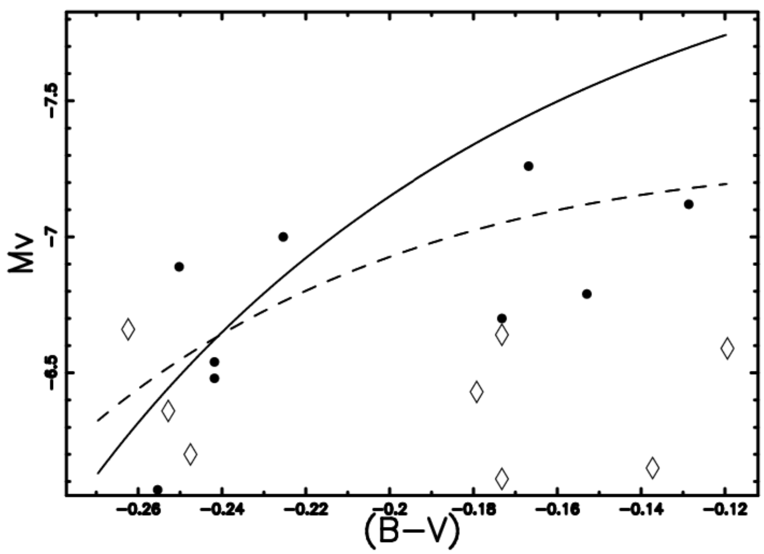

Figure 1: Theoretical visual magnitude ,$M_{\rm V}$, against $(B-V)_0$ for Galactic early B supergiants : Ia (full line) and Ib (dotted line). The observed values of magnitude as extracted from Table 3 of Searle 2008 are also reported : Ia (full circle) and Ib (empty diamond).

and

$$M_V = -31.38 + 2.623\,\ln\left(4012.0\,\left((B-V)_0 + 0.3961\right)^{-1}\right) + 7.870\,(B-V)_0 \quad (10)$$
$$supergiants\ Ib\ when\ 0.25 < (B-V)_0 < -0.14 \quad .$$

Figure 1 reports the theoretical visual magnitude , M_V , for the two classes here considered as well as the observational points as extracted from Table 3 of Searle et al. (2008).

The mass $\frac{\mathcal{M}}{\mathcal{M}_{\odot}}$ has expression

$$\frac{\mathcal{M}}{\mathcal{M}_{\odot}} = 10.0^{-3.713+0.5070\,\ln\left(4012.0\,((B-V)_0+0.3961)^{-1}\right)} \quad (11)$$
$$supergiants\ Ia\ when\ 0.25 < (B-V)_0 < -0.14 \quad ,$$

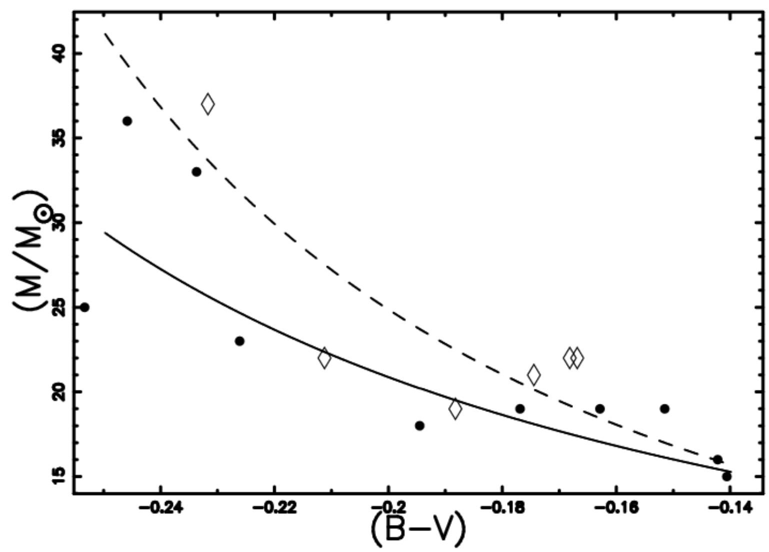

Figure 2: Theoretical mass , $\frac{\mathcal{M}}{\mathcal{M}_{\odot}}$, against $(B-V)_0$ for Galactic early B supergiants : Ia (full line) and Ib (dotted line). The observed values of mass as extracted from Table 4 of Searle 2008 are also reported : Ia (full circle) and Ib (empty diamond).

and

$$
\frac{\mathcal{M}}{\mathcal{M}_{\odot}} = 10.0^{-6.025+0.7476\ \ln\left(4012.0\,((B-V)_0+0.3961)^{-1}\right)} \tag{12}
$$
$$
supergiants\ Ib\ when\ 0.25 < (B-V)_0 < -0.14 \quad .
$$

Figure 2 reports the theoretical mass , $\frac{\mathcal{M}}{\mathcal{M}_{\odot}}$, for the two classes here considered as well as the observational points as extracted from Table 4 of Searle et al. (2008).

The radius , $\frac{R}{R_{\odot}}$, has expression

$$
\frac{R}{R_{\odot}} = 10.0^{8.733-0.7151\ \ln\left(4012.0\,((B-V)_0+0.3961)^{-1}\right)} \tag{13}
$$
$$
supergiants\ Ia\ when\ 0.25 < (B-V)_0 < -0.14 \quad ,
$$

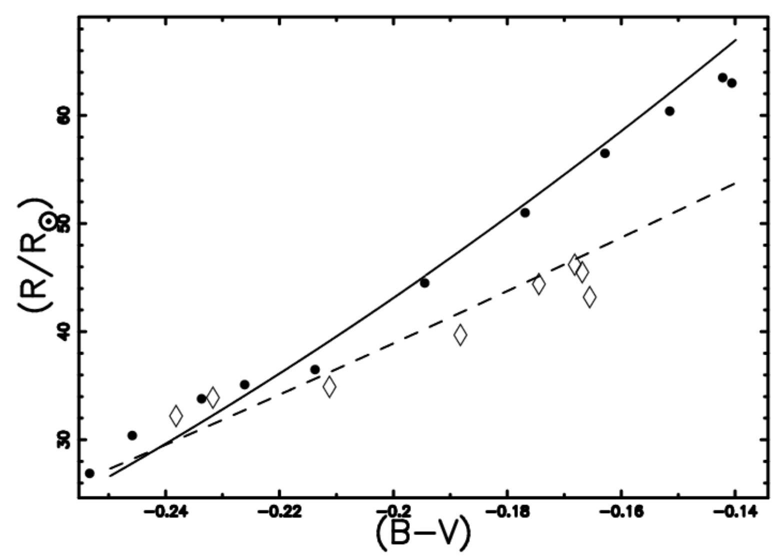

Figure 3: Theoretical radius , $\frac{R}{R_\odot}$, against $(B-V)_0$ for Galactic early B supergiants : Ia (full line) and Ib (dotted line). The observed value of radius as extracted from Table 4 of Searle 2008 are also reported : Ia (full circle) and Ib (empty diamond).

and

$$\frac{R}{R_\odot} = 10.0^{6.795-0.5245\ \ln\left(4012.0\,((B-V)_0+0.3961)^{-1}\right)} \qquad (14)$$
$$supergiants\ Ib\ when\ 0.25 < (B-V)_0 < -0.14 \quad .$$

Figure 3 reports the theoretical radius , $\frac{R}{R_\odot}$, for the two classes here considered as well as the observational points as extracted from Table 4 of Searle et al. (2008).

The logarithm of the luminosity , $\log_{10}(\frac{L}{L\odot})$, is

$$\log_{10}(\frac{L}{L\odot}) = 2.421 + 0.3068\ \ln\left(4012.0\ ((B-V)_0 + 0.3961)^{-1}\right) \qquad (15)$$
$$supergiants\ Ia\ when\ 0.25 < (B-V)_0 < -0.14 \quad ,$$

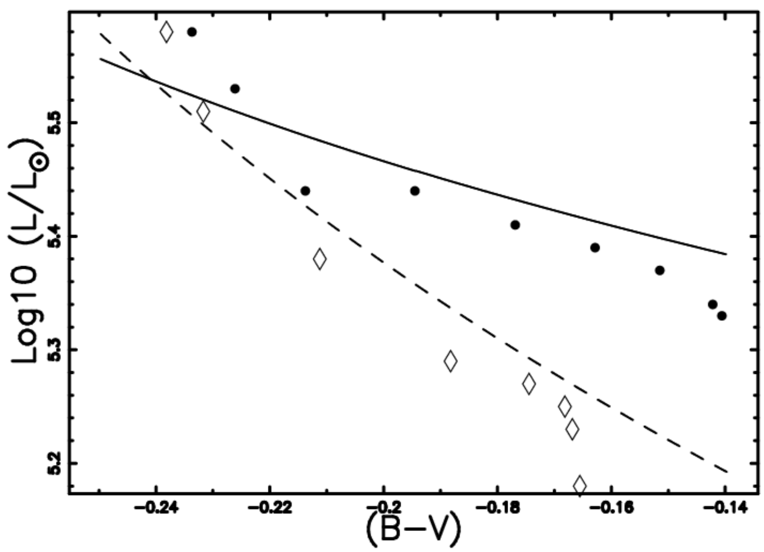

Figure 4: Theoretical logarithm of the luminosity , $\log_{10}(\frac{L}{L\odot})$, against $(B-V)_0$ for Galactic early B supergiants : Ia (full line) and Ib (dotted line). The observed values of luminosity as extracted from Table 4 of Searle 2008 are also reported : Ia (full circle) and Ib (empty diamond).

and

$$\log_{10}(\frac{L}{L\odot}) = -1.454 + 0.6882 \ln\left(4012.0\ \left((B-V)_0 + 0.3961\right)^{-1}\right) \tag{16}$$
$$\mathit{supergiants\ Ib\ when}\ 0.25 < (B-V)_0 < -0.14 \quad .$$

Figure 4 reports the theoretical logarithm of the luminosity , $\log_{10}(\frac{L}{L\odot})$, for the two classes here considered as well as the observational points as extracted from Table 4 of Searle et al. (2008).

The logarithm of the surface gravity , $\log_{10}(g[cgs])$, is

$$\log_{10}(g[cgs]) =$$
$$-10.61 + 0.4343 \ln\left(3.118\ \left(\left((B-V)_0 + 0.3961\right)^{-1}\right)^{1.167}\right)$$
$$-0.8686 \ln\left(0.00001891\ \left(\left((B-V)_0 + 0.3961\right)^{-1}\right)^{-1.646}\right) \tag{17}$$
$$\mathit{supergiants\ Ia\ when}\ 0.25 < (B-V)_0 < -0.14 \quad ,$$

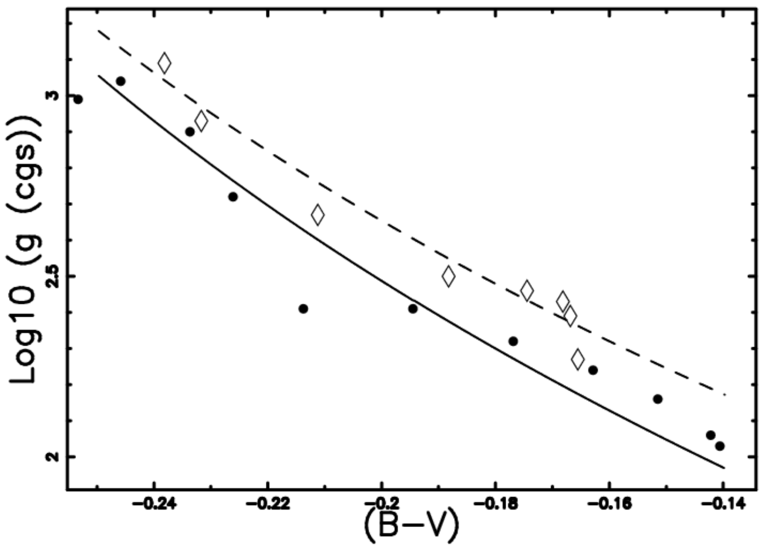

Figure 5: Theoretical logarithm of the surface gravity , $\log_{10}(g[cgs])$, against $(B-V)_0$ for Galactic early B supergiants : Ia (full line) and Ib (dotted line). The observed values of surface gravity as extracted from Table 4 of Searle 2008 are also reported : Ia (full circle) and Ib (empty diamond).

and

$$
\begin{gathered}
\log_{10}(g[cgs]) = \\
-10.61 + 0.4343 \ln \left(1.506 \left(((B-V)_0 + 0.3961)^{-1}\right)^{1.721}\right) \\
-0.8686 \ln \left(0.000008342 \left(((B-V)_0 + 0.3961)^{-1}\right)^{-1.2077}\right) \\
\textit{supergiants Ib when } 0.25 < (B-V)_0 < -0.14 \quad .
\end{gathered}
\tag{18}
$$

Figure 5 reports the theoretical logarithm of the surface gravity for the two classes here considered as well as the observational points as extracted from Table 4 of Searle et al. (2008). From a practical point of view, ϵ , the percentage of reliability of our results can also be introduced,

$$
\epsilon = (1 - \frac{|(F_{obs} - F_{num})|}{F_{obs}}) \cdot 100 \,, \tag{19}
$$

where F_{obs} is one of the fundamental parameter as given by the astronomical observations , and F_{num} the analogous fundamental parameter as given by our numerical relationships. The minimum , averaged and maximum efficiency in reproducing the observed fundamental parameters (respectively ϵ_{min} , $\overline{\epsilon}$ and ϵ_{max}) as given by formula (19) are reported in Table 3.

Table 3: Efficiency in deriving the fundamental parameters for B0-B5 supergiants

	Ia			Ib		
Fundamental Parameter	ϵ_{min} (%)	$\overline{\epsilon}$ (%)	ϵ_{max} (%)	ϵ_{min} (%)	$\overline{\epsilon}$ (%)	ϵ_{max} (%)
Visual magnitude	88.8	94.5	98.7	83.5	91.4	97.5
Mass	10.5	81.2	97.7	55.2	81.4	95.9
Radius	91.5	95.9	98.5	90.4	94.8	98.9
Logarithm Luminosity	98.7	99.2	99.6	98.3	99.1	99.6
Logarithm surface gravity	90.9	96.8	98.9	95.9	98.2	99.5

4 Conclusions

The eight parameters conjecture which allows to model five fundamental parameters of the stars is connected with the availability of calibration data on mass and luminosity as a function of the temperature or the intrinsic , unreddened color index $(B-V)_0$. Table 3 and 4 in Searle et al. (2008) offer both the splitting of early supergiants in two classes and a good coverage of the fundamental parameters in the range 15200 $K < T <$ 30000 K . The five fundamental parameters here analyzed can be parametrized as function of $(B-V)_0$ and Table 3 reports the efficiency of the model here implemented. As an example the two relationships (10) and (11) that regulates the visual magnitude have an averaged accuracy of 0.34 mag in Ia and 0.5 mag in Ib. This paper also parametrizes the logarithm of the surface gravity , $\log_{10}(g[cgs])$, as a function $(B-V)_0$, see formulas (6), (18), (19) and Figure 5. This parametrization allows to casts doubt of the fits of T that contain $\log_{10}(g[cgs])$ as a parameter , see for example Sekiguchi & Fukugita

(2000); Kovtyukh et al. (2008) , because the reverse is true: $\log_{10}(g[cgs])$ is a function of T or its observational counterpart $(B-V)_0$.

Acknowledgements I thank Lorenzo Ducci who drew my attention to Table 5 in Searle et al. (2008).

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